Math III Review Rational and Radical Equations

Simplifying Rational and Radical Equations

Ex/ Solve $2x = \sqrt{5x - 1} + 1$

Step 1: Subtract 1 from each side to isolate the radical term

$$2x - 1 = \sqrt{5x - 1}$$

Step 2: Square both sides

$$4x^2 - 4x + 1 = 5x - 1$$

Step 3: Set the right side equal to zero

$$4x^2 - 9x + 2 = 0$$

Step 4: Solve for x (factoring, quadratic formula, graphing)

$$x = \frac{1}{4} \quad and \quad x = 2$$

Step 5: Plug answers back into the original equation and check for extraneous solutions

$$2\left(\tfrac{1}{4}\right) = \sqrt{5\left(\tfrac{1}{4}\right) - 1} + 1 \qquad\qquad 2(2) = \sqrt{5(2) - 1} + 1$$

$$\tfrac{1}{2} \neq 1\tfrac{1}{2} \qquad\qquad\qquad\qquad 4 = 4$$

So $x = \frac{1}{4}$ is **not** a solution   So x = 2 is a solution

The solution $\frac{1}{4}$ is an **extraneous solution** because is a solution to the transformed equation, not to the original equation

Ex/ Solve $\frac{x}{x-1} - 1 = \frac{x}{2}$

Step 1: Get a common denominator, in this case $2(x-1)$

$$\frac{2x}{2(x-1)} - \frac{2(x-1)}{2(x-1)} = \frac{x(x-1)}{2(x-1)}$$

Step 2: Since the denominators are the same we only need to simplify the numerator

$$2x - 2(x-1) = x(x-1)$$

$$2x - 2x + 2 = x^2 - x$$

$$0 = x^2 - x - 2$$

Step 3: Solve for x

$$0 = (x-2)(x+1)$$

So $x = 2$ and $x = -1$

Step 4: Plug answers back into the original equation and check for extraneous solutions

$$\frac{2}{2-1} - 1 = \frac{2}{2} \qquad\qquad \frac{-1}{-1-1} - 1 = \frac{-1}{1}$$

$$1 = 1 \qquad\qquad\qquad -\frac{1}{2} \neq -1$$

So x = 2 is a solution   So x = -1 is **not** a solution

The solution -1 is an **extraneous solution**

Examples:

1. Solve for x: $\left(\frac{x+1}{5} - 2 = \frac{-4}{x}\right)^{5X}$

   A. x = 4
   B. x = 5
   C. x = 4, 5
   D. no solution

$x^2 + x - 10x = -20$
$ +20 \quad +20$
$x^2 - 9x + 20 = 0$
$(x-5)(x-4) = 0$

$x = 5, 4$
when they are plugd
in both work

2. Solve for x: $\left(\dfrac{8}{x-5} - \dfrac{9}{x-4} = \dfrac{5}{x^2-9x+20} \atop (x-5)(x-4) \right)^{(x-5)(x-4)}$

$(8x-32) - (9x-45) = 5$

$\quad\quad -x + 13 = 5$

$\quad\quad\quad -13 \quad -13$

$\quad\quad\quad\quad -x = -8$

$\quad\quad\quad\quad x = 8$

when plugued in $\boxed{x = 8}$ works

3. Solve for x: $8 - \sqrt{x+12} = 3$

$\quad -8 \quad\quad\quad -8$

$\quad\quad \dfrac{-\sqrt{x+12}}{-1} = \dfrac{-5}{-1}$

$\quad\quad \sqrt{x+12}^2 = 5^2$

$\quad\quad x + 12 = 25$

$\quad\quad -12 \quad -12$

$\quad\quad \boxed{x = 13}$ when plugued in
$\quad\quad\quad\quad\quad\quad x = 13$ works

4. Solve for x: $\left(\sqrt{x+15}\right)^2 = \left(5 + \sqrt{x}\right)^2$

$x + 15 = 25 + 10\sqrt{x} + x$

$-x$ $\quad\quad\quad\quad\quad\quad -x$

$\quad 15 = 25 + 10\sqrt{x}$

$\quad -25 \quad -25$

$\quad \dfrac{-10}{10} = \dfrac{10\sqrt{x}}{10}$

$\quad \sqrt{x} = -1$

$\quad\quad NS$